INTEGRATION TABLES

<u>Forms involving</u> u^n

1. $\displaystyle \int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$

2. $\displaystyle \int \frac{1}{u} \, du = \ln|u| + C$

<u>Forms involving</u> $a + bu$

3. $\displaystyle \int \frac{u}{a+bu} \, du = \frac{1}{b^2}\left(bu - a\ln|a+bu|\right) + C$

4. $\displaystyle \int \frac{u}{(a+bu)^2} \, du = \frac{1}{b^2}\left(\frac{a}{a+bu} + \ln|a+bu|\right) + C$

5. $\displaystyle \int \frac{u}{(a+bu)^n} \, du = \frac{1}{b^2}\left[\frac{-1}{(n-2)(a+bu)^{n-2}} + \frac{a}{(n-1)(a+bu)^{n-1}}\right] + C, \quad n \neq 1,2$

6. $\displaystyle \int \frac{u^2}{a+bu} \, du = \frac{1}{b^3}\left(-\frac{bu}{2}(2a-bu) + a^2\ln|a+bu|\right) + C$

7. $\displaystyle \int \frac{u^2}{(a+bu)^2} \, du = \frac{1}{b^3}\left(bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$

8. $\displaystyle \int \frac{u^2}{(a+bu)^3} \, du = \frac{1}{b^3}\left(\frac{2a}{a+bu} - \frac{a^2}{2(a+bu)^2} + \ln|a+bu|\right) + C$

9. $\displaystyle \int \frac{u^2}{(a+bu)^n} \, du = \frac{1}{b^3}\left[\frac{-1}{(n-3)(a+bu)^{n-3}} + \frac{2a}{(n-2)(a+bu)^{n-2}} - \frac{a^2}{(n-1)(a+bu)^{n-1}}\right] + C \quad n \neq 1,2,3$

10. $\displaystyle \int \frac{1}{u(a+bu)} \, du = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

11. $\displaystyle \int \frac{1}{u(a+bu)^2} \, du = \frac{1}{a}\left(\frac{1}{a+bu} + \frac{1}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

12. $\displaystyle \int \frac{1}{u^2(a+bu)} \, du = -\frac{1}{a}\left(\frac{1}{u} + \frac{b}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

13. $\displaystyle \int \frac{1}{u^2(a+bu)^2} \, du = -\frac{1}{a^2}\left(\frac{a+2bu}{u(a+bu)} + \frac{2b}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

<u>Forms involving</u> $a + bu + cu^2, \ b^2 \neq 4ac$

14. $\displaystyle \int \frac{1}{a+bu+cu^2} \, du = \begin{cases} \dfrac{2}{\sqrt{4ac-b^2}}\arctan\dfrac{2cu+b}{\sqrt{4ac-b^2}} + C, & b^2 < 4AC \\[3mm] \dfrac{1}{\sqrt{b^2-4ac}}\ln\left|\dfrac{2cu+b-\sqrt{b^2-4ac}}{2cu+b+\sqrt{b^2-4ac}}\right| + C, & b^2 > 4ac \end{cases}$

15. $\displaystyle \int \frac{1}{a+bu+cu^2} \, du = \frac{1}{2c}\left(\ln|a+bu+cu^2| - b\int \frac{1}{a+bu+cu^2} \, du\right)$

Forms involving $\sqrt{a+bu}$

16. $\displaystyle\int u^n \sqrt{a+bu}\,du = \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2} - na\int u^{n-1}\sqrt{a+bu}\,du\right]$

17. $\displaystyle\int \frac{1}{u\sqrt{a+bu}}\,du = \begin{cases} \dfrac{1}{\sqrt{a}}\ln\left|\dfrac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C, & 0 < a \\[3mm] \dfrac{2}{\sqrt{-a}}\arctan\sqrt{\dfrac{a+bu}{-a}} + C, & a < 0 \end{cases}$

18. $\displaystyle\int \frac{1}{u^n\sqrt{a+bu}}\,du = \frac{-1}{a(n-1)}\left[\frac{\sqrt{a+bu}}{u^{n-1}} + \frac{(2n-3)b}{2}\int\frac{1}{u^{n-1}\sqrt{a+bu}}\,du\right], \quad n \neq 1$

19. $\displaystyle\int \frac{\sqrt{a+bu}}{u}\,du = 2\sqrt{a+bu} + a\int\frac{1}{u\sqrt{a+bu}}\,du$

20. $\displaystyle\int \frac{\sqrt{a+bu}}{u^n}\,du = \frac{-1}{a(n-1)}\left[\frac{(a+bu)^{3/2}}{u^{n-1}} + \frac{(2n-5)b}{2}\int\frac{\sqrt{a+bu}}{u^{n-1}}\,du\right], \quad n \neq 1$

21. $\displaystyle\int \frac{u}{\sqrt{a+bu}}\,du = \frac{-2(2a-bu)}{3b^2}\sqrt{a+bu} + C$

22. $\displaystyle\int \frac{u^n}{\sqrt{a+bu}}\,du = \frac{2}{(2n+1)b}\left(u^n\sqrt{a+bu} - na\int\frac{u^{n-1}}{\sqrt{a+bu}}\,du\right)$

Forms involving $a^2 \pm u^2,\ 0 < a$

23. $\displaystyle\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\frac{u}{a} + C$

24. $\displaystyle\int \frac{1}{u^2-a^2}\,du = -\int\frac{1}{a^2-u^2}\,du = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

25. $\displaystyle\int \frac{1}{\left(a^2\pm u^2\right)^n}\,du = \frac{1}{2a^2(n-1)}\left[\frac{u}{\left(a^2\pm u^2\right)^{n-1}} + (2n-3)\int\frac{1}{\left(a^2\pm u^2\right)^{n-1}}\,du\right], \quad n \neq 1$

Forms involving $\sqrt{u^2 \pm a^2},\ 0 < a$

26. $\displaystyle\int \sqrt{u^2\pm a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2\pm a^2} \pm a^2\ln\left|u+\sqrt{u^2\pm a^2}\right|\right) + C$

27. $\displaystyle\int u^2\sqrt{u^2\pm a^2}\,du = \frac{1}{8}\left[u\left(2u^2\pm a^2\right)\sqrt{u^2\pm a^2} - a^4\ln\left|u+\sqrt{u^2\pm a^2}\right|\right] + C$

28. $\displaystyle\int \frac{\sqrt{u^2+a^2}}{u}\,du = \sqrt{u^2+a^2} - a\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C$

29. $\displaystyle\int \frac{\sqrt{u^2-a^2}}{u}\,du = \sqrt{u^2-a^2} - a\,\text{arcsec}\frac{|u|}{a} + C$

30. $\displaystyle\int \frac{\sqrt{u^2\pm a^2}}{u^2}\,du = \frac{-\sqrt{u^2\pm a^2}}{u} + \ln\left|u+\sqrt{u^2\pm a^2}\right| + C$

31. $\displaystyle\int \frac{1}{\sqrt{u^2\pm a^2}}\,du = \ln\left|u+\sqrt{u^2\pm a^2}\right| + C$

32. $\int \dfrac{1}{u\sqrt{u^2 + a^2}}\, du = \dfrac{-1}{a}\ln\left|\dfrac{a + \sqrt{u^2 + a^2}}{u}\right| + C$

33. $\int \dfrac{1}{u\sqrt{u^2 - a^2}}\, du = \dfrac{1}{a}\operatorname{arcsec}\dfrac{|u|}{a} + C$

34. $\int \dfrac{u^2}{\sqrt{u^2 \pm a^2}}\, du = \dfrac{1}{2}\left(u\sqrt{u^2 \pm a^2} \mp a^2 \ln\left|u + \sqrt{u^2 \pm a^2}\right|\right) + C$

35. $\int \dfrac{1}{u^2\sqrt{u^2 \pm a^2}}\, du = \mp\dfrac{\sqrt{u^2 \pm a^2}}{a^2 u} + C$

36. $\int \dfrac{1}{\left(u^2 \pm a^2\right)^{3/2}}\, du = \dfrac{\pm u}{a^2\sqrt{u^2 \pm a^2}} + C$

Forms involving $\boxed{\sqrt{a^2 - u^2},\ 0 < a}$

37. $\int \sqrt{a^2 - u^2}\, du = \dfrac{1}{2}\left(u\sqrt{a^2 - u^2} + a^2 \arcsin\dfrac{u}{a}\right) + C$

38. $\int u^2\sqrt{a^2 - u^2}\, du = \dfrac{1}{8}\left[u\left(2u^2 - a^2\right)\sqrt{a^2 - u^2} + a^4 \arcsin\dfrac{u}{a}\right] + C$

39. $\int \dfrac{\sqrt{a^2 - u^2}}{u}\, du = \sqrt{a^2 - u^2} - a\ln\left|\dfrac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

40. $\int \dfrac{\sqrt{a^2 - u^2}}{u^2}\, du = \dfrac{-\sqrt{a^2 - u^2}}{u} - \arcsin\dfrac{u}{a} + C$

41. $\int \dfrac{1}{\sqrt{a^2 - u^2}}\, du = \arcsin\dfrac{u}{a} + C$

42. $\int \dfrac{1}{u\sqrt{a^2 - u^2}}\, du = \dfrac{-1}{a}\ln\left|\dfrac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

43. $\int \dfrac{u^2}{\sqrt{a^2 - u^2}}\, du = \dfrac{1}{2}\left(-u\sqrt{a^2 - u^2} + a^2 \arcsin\dfrac{u}{a}\right) + C$

44. $\int \dfrac{1}{u^2\sqrt{a^2 - u^2}}\, du = \dfrac{-\sqrt{a^2 - u^2}}{a^2 u} + C$

45. $\int \dfrac{1}{\left(a^2 - u^2\right)^{3/2}}\, du = \dfrac{u}{a^2\sqrt{a^2 - u^2}} + C$

Forms involving $\boxed{\sin u\ \text{or}\ \cos u}$

46. $\int \sin u\, du = -\cos u + C$

47. $\int \cos u\, du = \sin u + C$

48. $\int \sin^2 u\, du = \dfrac{1}{2}(u - \sin u \cos u) + C$

49. $\int \cos^2 u\, du = \dfrac{1}{2}(u + \sin u \cos u) + C$

50. $\displaystyle\int \sin^n u \, du = -\frac{\sin^{n-1} u \cos u}{n} + \frac{n-1}{n}\int \sin^{n-2} u \, du$

51. $\displaystyle\int \cos^n u \, du = \frac{\cos^{n-1} u \sin u}{n} + \frac{n-1}{n}\int \cos^{n-2} u \, du$

52. $\displaystyle\int u \sin u \, du = \sin u - u \cos u + C$

53. $\displaystyle\int u \cos u \, du = \cos u + u \sin u + C$

54. $\displaystyle\int u^n \sin u \, du = -u^n \cos u + n\int u^{n-1} \cos u \, du$

55. $\displaystyle\int u^n \cos u \, du = u^n \sin u - n\int u^{n-1} \sin u \, du$

56. $\displaystyle\int \frac{1}{1 \pm \sin u} \, du = \tan u \mp \sec u + C$

57. $\displaystyle\int \frac{1}{1 \pm \cos u} \, du = -\cot u \pm \csc u + C$

58. $\displaystyle\int \frac{1}{\sin u \cos u} \, du = \ln|\tan u| + C$

Forms involving $\boxed{\tan u, \ \cot u, \ \sec u, \ \csc u}$

59. $\displaystyle\int \tan u \, du = -\ln|\cos u| + C$

60. $\displaystyle\int \cot u \, du = \ln|\sin u| + C$

61. $\displaystyle\int \sec u \, du = \ln|\sec u + \tan u| + C$

62. $\displaystyle\int \csc u \, du = \ln|\csc u - \cot u| + C$

63. $\displaystyle\int \tan^2 u \, du = -u + \tan u + C$

64. $\displaystyle\int \cot^2 u \, du = -u - \cot u + C$

65. $\displaystyle\int \sec^2 u \, du = \tan u + C$

66. $\displaystyle\int \csc^2 u \, du = -\cot u + C$

67. $\displaystyle\int \tan^n u \, du = \frac{\tan^{n-1} u}{n-1} - \int \tan^{n-2} u \, du, \quad n \neq 1$

68. $\displaystyle\int \cot^n u \, du = -\frac{\cot^{n-1} u}{n-1} - \int \cot^{n-2} u \, du, \quad n \neq 1$

69. $\displaystyle\int \sec^n u \, du = \frac{\sec^{n-2} u \tan u}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} u \, du, \quad n \neq 1$

70. $\displaystyle\int \csc^n u \, du = -\frac{\csc^{n-2} u \cot u}{n-1} + \frac{n-2}{n-1}\int \csc^{n-2} u \, du, \quad n \neq 1$

71. $\displaystyle\int \frac{1}{1 \pm \tan u} \, du = \frac{1}{2}\left(u \pm \ln|\cos u \pm \sin u|\right) + C$

72. $\displaystyle\int \frac{1}{1 \pm \cot u} \, du = \frac{1}{2}\left(u \mp \ln|\sin u \pm \cos u|\right) + C$

73. $\displaystyle\int \frac{1}{1 \pm \sec u} \, du = u + \cot u \mp \csc u + C$