AITKEN'S ALGORITHM

Step 1: Select x_0
Step 2: Calculate

$$x_1 = g(x_0), \quad x_2 = g(x_1)$$

Step3: Calculate

$$x_3 = x_2 + \frac{\lambda_2}{1 - \lambda_2}\left[x_2 - x_1\right], \qquad \lambda_2 = \frac{x_2 - x_1}{x_1 - x_0}$$

Step 4: Calculate

$$x_4 = g(x_3), \quad x_5 = g(x_4)$$

and calculate x_6 as the extrapolate of $\{x_3, x_4, x_5\}$. Continue this procedure, ad infinatum.

Of course in practice we will have some kind of error test to stop this procedure when believe we have sufficient accuracy.

EXAMPLE

Consider again the iteration

$$x_{n+1} = 6.28 + \sin(x_n), \qquad n = 0, 1, 2, ...$$

for solving

$$x = 6.28 + \sin x$$

Now we use the Aitken method, and the results are shown in the accompanying table. With this we have

$$\alpha - x_3 = 7.98 \times 10^{-4}, \quad \alpha - x_6 = 2.27 \times 10^{-6}$$

In comparison, the original iteration had

$$\alpha - x_6 = 1.23 \times 10^{-2}$$

GENERAL COMMENTS

Aitken extrapolation can greatly accelerate the convergence of a linearly convergent iteration

$$x_{n+1} = g(x_n)$$

This shows the power of understanding the behaviour of the error in a numerical process. From that understanding, we can often improve the accuracy, thru extrapolation or some other procedure.

This is a justification for using mathematical analyses to understand numerical methods. We will see this repeated at later points in the course, and it holds with many different types of problems and numerical methods for their solution.