False-Position (*Regula Falsi*) Method

The approach here is to estimate the root by finding the point at which a line drawn between x_l and x_u crosses the x axis. That is, you are performing *linear interpolation* between x_l and x_u to find the approximate root. Like the bisection method, the false-position method will always converge, and generally it will converge faster than the bisection method. Unfortunately, this general rule is not *always* true, and sometimes the bisection method converges faster than the false-position method.

To get the update formula for the false-position method, recall that the equation for the line between x_l and x_u is given by

$$y = f(x_u) + \frac{f(x_l) - f(x_u)}{x_l - x_u}(x - x_u).$$

Hence, to find the intersection of this line with the *x* axis, we must solve the equation

$$0 = f(x_u) + \frac{f(x_l) - f(x_u)}{x_l - x_u}(x_r - x_u).$$

This leads directly to the update formula

$$x_r = x_u - f(x_u)\frac{x_l - x_u}{f(x_l) - f(x_u)}$$

for the false-position method. The points x_l and x_u are updated using the same rule used for the bisection method.