

# study unit fourteen

hypothesis tests applied to means: two related samples

**W**e have already discussed hypothesis testing in study unit 13. In this study unit we shall test a hypothesis empirically in order to determine scientifically whether there is a significant difference between the means of two sets of scores. This is something that we often have to do in the work situation — determine the difference between two sets of data. Remember the example of the height of all men and women in South Africa in study unit 13?

There are different kinds of test statistics that we can test by means of hypotheses, depending on the type of data. In this study unit we look at only one type of test statistic: the  $t$ -test for two related groups.



Use the schematic representation on the previous page and write down what hypothesis tests we shall be doing next. Also take another look at the tree diagram in Howell. Check what you write below against this diagram.

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

How many did you find? You should have written down four test statistics.

What are related samples? \_\_\_\_\_

In this study unit we deal only with the first type, namely the statistic for two related samples. But what does that mean? Are they the same as independent samples? When do we speak of related samples and when do we refer to independent samples? This distinction will be clear to you once you have

completed this study unit and the next one. After that we shall look at the other types of test statistics.



To explain the concept of related samples as clearly as possible, Howell uses good, understandable examples. Read through his introduction and section 13.1.

- Make sure that you understand what is meant by related samples.
- Use the knowledge you have acquired to do the following exercises:

	True	False
1 Here we are working with two sets of data on the same participants (ie persons who form part of the sample).	<input type="checkbox"/>	<input type="checkbox"/>
2 We expect the two sets of figures (variables) to correlate.	<input type="checkbox"/>	<input type="checkbox"/>
3 We take correlation into account when computing the <i>t</i> -statistic.	<input type="checkbox"/>	<input type="checkbox"/>
4 The only way to obtain related samples is when the same participant contributes two scores.	<input type="checkbox"/>	<input type="checkbox"/>

- 
- 1 true
  - 2 true
  - 3 false
  - 4 false

Let us explain in greater detail what is meant by related samples, for you have to be able to tell when a research problem or a research hypothesis entails related or independent samples. Since participants can be matched on the basis of common attributes, you should combine the discussion below with Howell's introduction to chapter 13 and his section 13.1 and make sure that you know the contents well.

We say that two groups of people or participants are matched according to a variable if for each member allocated to one group, the other group is assigned a member who corresponds with the member of group one in respect of the particular variable. Thus participant pairs are selected in such a way that the two members of each pair are as similar as possible in respect of the relevant variable(s). Variables like intelligence, age, height, eye colour and gender can be used to match persons or participants.

**To sum up: the only ways of obtaining related sample scores are**

- by matching
- when one participant contributes two scores

How do we compute the test statistic for related samples? \_\_\_\_\_

Now that you know what a related sample is, the next step is to determine

empirically whether there is a significant difference between two related samples. This involves a fairly simple computation by means of a specific formula. But before you can apply the formula you first have to know how to compute difference scores. To do this you must know how to compute the mean ( $\bar{D}$ ) and the standard deviation ( $s_D$ ) of difference scores.



These are not new formulas. They are the same formulas that you used to compute the mean in study unit 7 and the standard deviation in study unit 8.



A practical example is always a useful learning experience, so it would be a good idea first to go through Howell's section 13.2 as an example of how to compute the  $t$ -test. If you are not sure how to compute the various steps of the formula, revise the formula for  $\bar{X}$  and  $s$  in study units 7 and 8 respectively. Howell provides the computed values of  $\bar{D}$  and  $s_D$ . Check them by using the relevant formulas.

Also try to follow the steps in hypothesis testing that you learned in study unit 13. Although less explicitly, Howell does follow the basic steps.

Complete the following:

- 1 A difference score is \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
- 2 Check the computation of the difference scores in the example. Can you see when the scores acquire positive or negative values?
- 3 Check the computation of the  $t$ -test in the example. Do you get the same answer?
- 4 Write down the formula for degrees of freedom.  
\_\_\_\_\_
- 5 Write down the information that you need to determine the critical value (one- or two-tailed test, significance level and number of degrees of freedom). Then find the value in the table and see if it is the same as Howell's.  
\_\_\_\_\_

Now that you have carefully checked a detailed example and have seen the various steps, let us test to see how well you understand it.



Before you do an exercise on your own, work through the complete example which follows so that you can see what we expect you to do in the different steps.

What follows is a fully worked-out practical example of how to do the  $t$ -test for related groups. The  $t$ -test is done according to the nine steps set out in study unit 13. Did you remember to write down the steps on a piece of cardboard and stick it above your desk? If not, do so now before you work through the example. You need to be comfortable with the logical flow of the steps and must be able to follow the steps in the exam so that you can use the relevant test statistic (in this case the  $t$ -test for related groups) and solve the problem statement statistically. These nine steps can be used in the same way for other test statistics (like the  $t$ -test for independent groups and also the  $F$ -test) — this is another good reason for familiarising yourself with the different steps now.

Work through the example and check all the calculations. Make sure that you understand all the steps in the process.

### Example of a $t$ -test for related groups

Prof Smith wants to determine whether there is a difference in the performance scores of a group of old ( $> 50$ ) and a group of young ( $< 30$ ) IOP201-Q students in an assignment question. He randomly selects 20 students after pairing them off on the basis of gender and age. He decides to test the hypothesis on both the 0,05 and 0,01 level of significance.

#### Data

Old	Young
4	5
3	8
7	6
6	6
5	6
4	4
5	7
4	7
3	6
2	4

Prof Smith follows these steps:

#### Step 1: Formulate the null hypothesis

In words

*There is no difference between the performance scores of old and young students.*

or

*Old and young students perform equally well.*

### In symbols

$$H_0 : \mu_O - \mu_Y = 0$$

or

$$H_0 : \mu_O = \mu_Y$$

### Step 2: Formulate the alternative hypothesis

In words

*There is a difference between the performance scores of old and young students.*

or

*Old and young students do not perform equally well.*

### In symbols

$$H_1 : \mu_O - \mu_Y \neq 0$$

or

$$H_1 : \mu_O \neq \mu_Y$$

### Step 3: Determine whether the test is one or two-tailed

In this example (like those you will have to handle) the decision whether to use a one or a two-tailed test is left to you. You therefore have to study the problem statement to see if it is directional or nondirectional. In the example we are working through, we are only investigating whether there is a difference between the means of the two groups and not whether the one group does better or worse than the other group. We are therefore working with a nondirectional hypothesis and choose the two-tailed test.

### Step 4: Determine the level of significance

Fortunately you will not have to make a decision here because we will always specify at what level of significance you must test the hypothesis. In this case we have specified two levels, namely 0,05 and 0,01.

### Step 5: Compute the test statistic

Given our problem statement (note that the two groups are *paired off*), we use the *t*-test for related groups and the following formula:

$$t = \frac{\bar{D}-0}{\frac{s_D}{\sqrt{N}}}$$

To use the formula, we first have to compute the difference scores (*D*). We do this by completing the table and then doing certain computations to obtain the means and the standard deviation. We do this as follows:

Old	Young	$D$	$D^2$
4	5	-1	1
3	8	-5	25
7	6	1	1
6	6	0	0
5	6	-1	1
4	4	0	0
5	7	-2	4
4	7	-3	9
3	6	-3	9
2	4	-2	4
		$\Sigma D = -16$	$\Sigma D^2 = 54$

$$\begin{aligned} \bar{D} &= \frac{\Sigma D}{N} \\ &= \frac{-16}{10} \\ &= -1,6 \end{aligned}$$

To use the formula for the  $t$ -test computation, we first have to compute the variance in the data. If you page back to study unit 8, you will see that we have already done so and used the following formula for the  $X$  values:

$$s_X^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N - 1}$$

In the case of the  $t$ -test for related groups, we have two sets of scores. The logical thing to do is to compute the difference between the two sets of scores and then get the variance in the difference scores.

The data in the  $D$  column ( $D$  for difference scores) are a set of  $X$  variables. You therefore simply replace the  $X$  in the formula with a  $D$ , like this:

$$\begin{aligned} s_D^2 &= \frac{\Sigma D^2 - \frac{(\Sigma D)^2}{N}}{N - 1} \\ &= \frac{54 - \frac{(-16)^2}{10}}{9} \\ &= \frac{54 - 25,6}{9} \\ &= \frac{28,4}{9} \\ &= 3,16 \end{aligned}$$

$$\begin{aligned} s_D &= \sqrt{s_D^2} \\ &= \sqrt{3,16} \\ &= 1,78 \end{aligned}$$

The standard deviation is therefore 1,78.

$$\begin{aligned}
t &= \frac{\bar{D} - 0}{\frac{s_D}{\sqrt{N}}} \\
&= \frac{-1,6 - 0}{\frac{1,78}{\sqrt{10}}} \\
&= \frac{-1,6}{\frac{1,78}{3,16}} \\
&= \frac{-1,6}{0,56} \\
&= -2,86
\end{aligned}$$

**Step 6: Determine the degrees of freedom**

Using the following formula:

$$\begin{aligned}
df &= N - 1 \\
&= 10 - 1 \\
&= 9
\end{aligned}$$

**Step 7: Determine the critical value**

Use table E.6 in Howell to read the critical values.

$$t_{0,05} (9) = 2,262$$

$$t_{0,01} (9) = 3,250$$

**Step 8: Decide whether the null hypothesis should be rejected/not rejected**

From the computations we have done and the tables, we have determined:

- the value of the test statistic is -2,86; and that
- the critical value at the 0,05 level is 2,262 and at the 0,01 level it is 3,250

Use the following decision-making rules:

- reject  $H_0$  if the test statistic  $>$  the critical value
- do not reject  $H_0$  if the test statistic  $<$  the critical value

$$t = -2,86 \quad \therefore |t| = 2,86 \quad (\text{Remember that we are working with absolute values and therefore ignore the minus sign.})$$

**For**  $\alpha = 0,05$

$$2,86 > 2,262$$

$\therefore$  We reject the null hypothesis.

**For**  $\alpha = 0,01$

$$2,86 < 3,250$$

$\therefore$  We cannot reject the null hypothesis. (Remember: It is unscientific and wrong to say that we accept the null hypothesis.)

### Step 9: Interpret the findings

Prof Smith can say with 95% certainty that there is a difference between the performance scores of old and young students in the assignment question, but he cannot say so with 99% certainty.

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Is it clear to you what we have done in the different steps? If not, go through the example again. If you are satisfied that you have a good idea of what you have to do in each step, go on to the activities which follow. You will have to concentrate harder because Howell's example is not worked out in as much detail.

Do section 13.3 on your own without looking at the different steps and see if you arrive at the same answer as Howell.

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Check your answer against Howell's.

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A further step in the process of hypothesis testing is to determine the number of degrees of freedom and read their critical value from table E.6. When you have managed this once, it is perfectly simple.



Study Howell's explanation in 13.2. Remember that we have been working with the number of *pairs* minus 1 degrees of freedom, since we are dealing with two *related* samples. Check the decision-making rules and follow Howell's argument for rejecting or not rejecting the null hypothesis.

Once you have done this, do the following exercises.

1 Test your ability to find the following critical values in the relevant table:

- 1.1 one-tailed test, 29*df*, 1% significance level
- 1.2 one-tailed test, 16*df*, 5% significance level
- 1.3 two-tailed test, 8*df*, 2% significance level
- 1.4 two-tailed test, 29*df*, 2% significance level
- 1.5 two-tailed test, 23*df*, 10% significance level

2 Would you reject  $H_0$  in the following circumstances? Why?

- 2.1  $t$ -test statistic = 2,302 and critical value = 2,508
  - 2.2  $t$ -test statistic = 2,682 and critical value = 3,792
  - 2.3  $t$ -test statistic = 3,248 and critical value = 2,704
- 

- 1.1 2,462
- 1.2 1,746
- 1.3 2,896

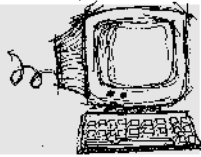


- 1.4 2,462
- 1.5 1,714
- 2.1 no
- 2.2 no
- 2.3 yes

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**You have now gone carefully through detailed examples of hypothesis tests with two related samples. You should be able to**

- explain what is meant by a related sample
- know *when* to use a *t*-test for related groups
- test a hypothesis by *applying* the *t*-test for related groups and *computing* a test statistic
- draw a final *conclusion* about your research hypothesis



Work through section 13.5 in Howell and make sure that you can identify the *t*-value on the printout.



We have now gone through the whole process of hypothesis testing when dealing with two related samples. For more experience in this kind of hypothesis testing, do the following exercises:

13.11 and 13.13

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Compare your answers with the model answers in the *Student handbook*.

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## Nonparametric equivalent of the *t*-test for related samples

Having read this heading, you must have noted the new term “nonparametric”. You will learn what a nonparametric statistical test is and how it differs from a parametric statistical test when you study the introduction to chapter 20 in Howell.

In this section we deal theoretically with the nonparametric equivalent of the *t*-test for related samples by looking at Wilcoxon’s matched-pairs signed-ranks test. As an industrial psychologist you have to know what to do when your data are not normally distributed. That is what we explain in this section.



Since you do not need to know the details of nonparametric equivalents, you only have to study the first paragraph of Howell's section 20.2 in order to complete the table below.

Complete the table partially by filling in the parametric and nonparametric tests for two related samples:

Parametric	Nonparametric equivalent
1 $t$ -test for two related samples	
2	
3	

Once you have completed study units 15 and 17, return to this table and complete it by adding the nonparametric equivalents of the parametric tests.

(Also take another look at the table that you completed at the beginning of this study unit.)

After completing the table you should be able to give the nonparametric equivalent of the  $t$ -test for related samples.

Exercises 1 and 2 and exercise 13.11 in Howell

**Data for exercise 1** (taken form the third edition of Howell's book)

As part of a study to reduce smoking, a national organization ran an advertising campaign to convince people to quit smoking. To evaluate the effectiveness of their campaign, they had 15 subjects record the average number of cigarettes smoked per day in the week before and the week after exposure to the ad.

The data follow:



Subject	Before	After
1	45	43
2	16	20
3	20	17
4	33	30
5	30	25
6	19	19
7	33	34
8	25	28
9	26	23
10	40	41
11	28	26
12	36	40
13	15	16
14	26	23
15	32	34

Run the appropriate  $t$ -test.

**Data for exercise 2** (taken from the third edition of Howell's book)

Assume the data in Exercise 1 had come out differently. The new data follow:

Subject	Before	After
1	45	59
2	16	35
3	20	30
4	33	40
5	30	20
6	19	10
7	33	20
8	25	20
9	26	36
10	40	46
11	28	10
12	36	25
13	15	8
14	26	35
15	32	46

Run the appropriate  $t$ -test.

The output for exercise 1 looks like this:

Variable	Obs	Mean	Std. Dev.
before	15	28.26667	8.622783
after	15	27.93333	8.811248
diff.	15	.3333333	2.870208

With computer program output you do not need to look up the critical value in the tables

Ho: diff = 0 (paired data)  
 $t = 0.45$  with 14 d.f.  
 Pr > |t| = 0.6597  
 95% conf. interval = (-1.2561, 1.9228)

The probability (alpha value) is given directly

Do no reject  $H_0$

Exercise 2:  $t = -0,35$

Exercise 13.11: see *Student handbook*